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Three perspectives on complexity: entropy, compression, subsymmetry

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Abstract. There is no single universally accepted definition of '*Complexity*'. There are several perspectives on complexity and what constitutes complex behaviour or complex systems, as opposed to regular, predictable behaviour and simple systems. In this paper, we explore the following perspectives on complexity: *effort-to-describe* (Shannon entropy H, Lempel-Ziv complexity LZ), *effort-to-compress* (ETC complexity) and *degree-of-order* (Subsymmetry or $SubSym$). While Shannon entropy and LZ are very popular and widely used, ETC is relatively a new complexity measure. In this paper, we also propose a novel normalized complexity measure $SubSym$ based on the existing idea of counting the number of subsymmetries or palindromes within a sequence. We compare the performance of these complexity measures on the following tasks: (A) characterizing complexity of short binary sequences of lengths 4 to 16, (B) distinguishing periodic and chaotic time series from 1D logistic map and 2D Hénon map, (C) analyzing the complexity of stochastic time series generated from 2-state Markov chains, and (D) distinguishing between tonic and irregular spiking patterns generated from the 'Adaptive exponential integrate-and-fire' neuron model. Our study reveals that each perspective has its own advantages and uniqueness while also having an overlap with each other.

1 Introduction

The seemingly naive procedure of finding decimal expansion of a real number, synchronized beating of heart cells, irregular firing of neurons in the brain, global economy and society, motion of planets and galaxies, the everyday weather and the Internet – are but a few examples of complex systems. While there is little agreement on the precise mathematical definition of *complexity*, there is no doubt on the ubiquity of complex systems in nature and society.

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The ability to model, analyze and understand complex systems is invaluable since it allows us to control, predict and exploit complex phenomena to our advantage. For example, by characterizing the complexity of the chaotic nature of cardiovascular dynamics, we may one day understand the effects of aging [1]. Understanding complexity may be the key to unlocking the mysteries of the brain since chaotic behaviour is abound in the central nervous system [2,3]. Complex behaviour of neural signals may even be necessary for multiplexing a large number of neural signals in the brain for successful communication in the presence of neural noise and interference [4]. Pseudo-random sequences having high linear (and nonlinear) complexity, but easy to generate in hardware, are employed in various cryptographic protocols [5]. Complexity and randomness is increasingly playing an important role in psychology to decipher behaviours and mental processes pertaining to memory (see [6] and references therein), cognitive effects of aging and human judgments of visual complexity [7]. Such a list is by no means exhaustive, but only serves to indicate the widespread prevalence of complex systems and use of the notion of complexity to characterize such systems.

There are broadly two categories of complex systems – deterministic and non-deterministic (stochastic).[1] Chaotic systems are examples of deterministic complex systems which are capable of producing rich and complex behaviour ranging from periodic to quasi-periodic to completely random-looking and unpredictable outputs. Stochastic systems are inherently non-deterministic ('random') and capable of exhibiting complex dynamics which can be predicted only probabilistically, at best. In either case, measuring complexity (of time series measurements from such systems) is an important step in characterizing or modeling the system/phenomenon under study. In this paper, we concern ourselves with the all-important question: *What is complexity?*

1.1 Complexity: many facets

There is no single universally accepted definition of '*Complexity*'. There are several perspectives on complexity and what constitutes complex behaviour or complex systems, as opposed to regular, predictable behaviour and simple systems. With the recent availability of high speed and low cost computational tools at our disposal, analysis of complex systems has become data-intensive. Lloyd [8] lists three questions which researchers ask in order to quantify the complexity of the thing (object) under study:

- How hard is it to describe? (*effort-to-describe*)
- How hard is it to create? (*effort-to-create*)
- What is its degree of organization? (*degree-of-order*).

We have indicated these questions with the corresponding italicized phrases in parenthesis (not present in Lloyd's original formulation). In this paper, we study three perspectives of complexity namely: *effort-to-describe*, *effort-to-compress* and *degree-of-order*. We have chosen a new perspective '*effort-to-compress*' instead of '*effort-to-create*' and our interpretation of *degree-of-order* is slightly different from Lloyd's (as will be evident in this paper). Our reasons for choosing these categories for characterizing complexity are as follows:

- Lloyd [8] rightly observes that though there is a multiplication of complexity measures, several of these measures represent variations of only a handful of

[1] Most naturally occurring systems are a hybrid since stochastic noise is inevitable.